$$\varphi'(z) = \exp\left[-\frac{F_s}{\pi i}\ln\frac{\sqrt{z^2-a^2}-\sqrt{b^2-a^2}}{\sqrt{z^2-a^2}+\sqrt{b^2-a^2}}\right],$$

while we obtain the following expression for the normal stress on the segment $[-a; a]$ between the slits

$$N(x)=\frac{2\mu\left[\exp 2F_s\left(1+\frac{1}{\pi}\operatorname{arctg}\frac{2\sqrt{(b^2-a^2)(a^2-x^2)}}{b^2-2a^2+x^2}\right)-1\right]}{1+\frac{\mu}{\lambda+\mu}\exp 2F_s\left(1+\frac{1}{\pi}\operatorname{arctg}\frac{2\sqrt{(b^2-a^2)(a^2-x^2)}}{b^2-2a^2+x^2}\right)}.$$

The linear dimension of the plastic region is determined by the formula

$$a=b\left[1-\frac{(1-\nu^2)^2P_0^2}{4E^2b^2F_s^2}\right]^{1/2},$$

where ν is the Poisson ratio; E is the Young modulus; the constant F_s is found from (1.15).

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ALGORITHM FOR STUDYING THE NONLINEAR DEFORMATION AND STABILITY OF CIRCULAR CYLINDRICAL SHELLS WITH INITIAL SHAPE FLAWS

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Axisymmetric deflections have been examined in most of the well-known solutions of problems concerning the stability of shells with initial deflections. Some of the studies have examined the effect of nonaxisymmetric deflections. The solutions have been obtained either in a classical formulation, without allowance for the moments of the initial stress state, or in a formulation which presumed the development of initial deflections, without restructuring, during nonlinear deformation under axisymmetric loads.

Below we obtain a fairly general solution to the problem, without restrictions on the load or the form of the initial and bifurcative deflections. We use the method of finite elements in displacements. The finite elements are chosen in the form of rectangles of natural curvature having form functions which consider their displacement as rigid bodies.

We will examine a circular cylindrical shell of the length L, radius R, and thickness h. The initial shape flaws are given either by the series $w^0 = \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}\cos i\varphi\cos j\pi x/L$, or by a two-dimensional set of nodal values of the initial deflection and its derivatives $\overline{w^0} =$

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$\{w_1, w_{\xi 1}, w_{\varphi 1}, \ldots, w_k, w_{\xi k}, w_{\varphi k}, \ldots, w_{\varphi n}\}$. Here, x and φ are the longitudinal and angular coordinates; N and M are the number of terms of the Fourier series in the expansion of the initial deflection w^0; n is the number of nodes of the theoretical finite-element grid; $\xi = x/R$, w_{ij} are the amplitudes of the initial deflection; ξ and φ in the subscripts denote differentiation.

The initial deflections are approximated on each finite element by a cubic polynomial whose unknown coefficients are expressed through the nodal values of the initial deflection and its derivatives. The shell is loaded by an arbitrary system of surface loads $q_i(x, y)$, linear contour forces $P_{ci}(x, y)$ and moments $M_{ci}(x, y)$, and local forces $P_{\ell i}$ and moments $M_{\ell i}$. Here, i = 1, 2, 3 corresponds to the directions of the axes x, y, and z.

1. Finite Element of the Shell. Using the solution [1], we write expressions for the displacements of the finite element

$$
\begin{gathered}
u = \alpha_1\xi\eta + \alpha_2\xi + \alpha_3\eta + \alpha_4 - \alpha_6 s - \alpha_{20}c, \\
v = \alpha_5\xi\eta + \alpha_6\xi c + \alpha_7\eta + \alpha_8 - \alpha_{20}\xi s + \alpha_{23}c - \alpha_{21}s, \\
w = \alpha_9\xi^3\eta^3 + \alpha_{10}\xi^3\eta^2 + \alpha_{11}\xi^3\eta + \alpha_{12}\xi^3 + \alpha_{13}\xi^2\eta^3 + \alpha_{14}\xi^2\eta^2 + \\
+ \alpha_{15}\xi^2\eta + \alpha_{16}\xi^2 + \alpha_{17}\xi\eta^3 + \alpha_{18}\xi\eta^2 + \alpha_{19}\xi\eta + \alpha_{20}\xi c + \alpha_{21}\eta^3 + \\
+ \alpha_{22}\eta^2 + \alpha_{23}s + \alpha_{24}c + \alpha_6\xi s, \ \xi = k_2x, \ \eta = k_2y, \\
c = \cos\varphi, \ s = \sin\varphi, \ u = k_2u', \ v = k_2v', \ w = k_2w', \ k_2 = R^{-1}.
\end{gathered}
$$

In matrix form, $\mathbf{u} = \mathbf{P}\boldsymbol{\alpha}$ ($\mathbf{u} = \{u, v, w\}$ is the vector of the displacements of points of the finite element; $\mathbf{P}$ is a 3 × 24 coupling matrix; $\boldsymbol{\alpha} = \{\alpha_1, \ldots, \alpha_{24}\}$ is the vector of the unknown coefficients of the polynomials).

We adopt the kinematic relations for the finite element in the form

$$
\begin{gathered}
\varepsilon_1 = u_\xi + w_\xi^0 w_\xi + \frac{1}{2}w_\xi^2, \quad \varepsilon_2 = v_\eta + w + w_\eta^0 w_\eta + \frac{1}{2}w_\eta^2, \\
\varepsilon_3 = u_\eta + v_\xi + w_\xi^0 w_\eta + w_\eta^0 w_\xi + w_\xi w_\eta, \\
\chi_1 = -w_{\xi\xi}k_2, \ \chi_2 = v_\eta - k_2 w_{\eta\eta}, \quad \chi_3 = 2(v_\xi - k_2 w_{\xi\eta})
\end{gathered}
$$

(the superscript 0 denotes the initial deflection; ξ and η henceforth denote differentiation with respect to ξ and η). In matrix form,

$$
\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_l + \boldsymbol{\varepsilon}_0 + \boldsymbol{\varepsilon}_n = (\mathbf{A}_l + \mathbf{A}_0 + \mathbf{A}_n)\mathbf{u}, \tag{1.1}
$$

where $\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \chi_1, \chi_2, \chi_3\}$ is the vector of the strains and the changes in curvature of points of the finite element

$$
\mathbf{A}_l^{\mathrm{T}} = \begin{vmatrix} (\ )_\xi & 0 & (\ )_\eta & 0 & 0 & 0 \\ 0 & (\ )_\eta & (\ )_\xi & 0 & (\ )_\eta & 2(\ )_\xi \\ 0 & (\ ) & 0 & -k_2(\ )_{\xi\xi} & -k_2(\ )_{\eta\eta} & -2k_2(\ )_{\xi\eta} \end{vmatrix};
$$

$$
\mathbf{A}_0^{\mathrm{T}} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ w_\xi^0(\ )_\xi & w_\eta^0(\ )_\eta & [w_\xi^0(\ )_\eta + w_\eta^0(\ )_\xi] & 0 & 0 & 0 \end{vmatrix};
$$

$$
\mathbf{A}_n^{\mathrm{T}} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2}(\ )_\xi^2 & \frac{1}{2}(\ )_\eta^2 & (\ )_\xi(\ )_\eta & 0 & 0 & 0 \end{vmatrix}.
$$

Using the solution in [2], we write expressions for the displacements, internal forces, and potential energy of the finite element

$$
\begin{gathered}
\mathbf{u} = \mathbf{P}_1\bar{\mathbf{u}}, \ \mathbf{P}_1 = \mathbf{P}\mathbf{B}^{-1}, \ \mathbf{T} = \mathbf{D}\boldsymbol{\varepsilon} = \mathbf{T}_l + \mathbf{T}_0 + \mathbf{T}_n = \\
= \mathbf{D}\boldsymbol{\varepsilon}_l + \mathbf{D}\boldsymbol{\varepsilon}_0 + \mathbf{D}\boldsymbol{\varepsilon}_n, \ \Pi_i = W_i - A_i, \\
W_i = \frac{1}{2}\iint_s \mathbf{T}^{\mathrm{T}}\boldsymbol{\varepsilon}\,ds, \quad A_i = \iint_s \mathbf{q}^{\mathrm{T}}\mathbf{u}\,ds + \int_l \mathbf{R}_c^{\mathrm{T}}\mathbf{u}_c\,dl + \mathbf{R}_\ell^{\mathrm{T}}\bar{\mathbf{u}}_\ell.
\end{gathered} \tag{1.2}
$$

Here, $\bar{\mathbf{u}} = \{u_i, v_i, w_i, w_{\xi i}, \theta_i, w_{\xi\eta i}, \ldots, w_{\xi\eta j}, \ldots, w_{\xi\eta c}, \ldots, w_{\xi\eta n}\}$ is the vector of the nodal displacements of the element; $\mathbf{B}$ is a 24×24 matrix; $\mathbf{T} = \{T_1, T_2, T_3, M_1, M_2, M_3\}$ is the vector of the internal force and moments; $\mathbf{q} = \{q_1, q_2, q_3\}$ is the vector of the external surface load; $\mathbf{R}_c = \{P_{c1}, P_{c2}, P_{c3}, M_{c1}, M_{c2}, M_{c3}\}$ and $\mathbf{R}_\ell = \{P_{\ell 1}, P_{\ell 2}, P_{\ell 3}, M_{\ell 1}, M_{\ell 2}, M_{\ell 3}\}$ are the vectors of the contour and local forces and moments; $_c = \{u, v, w, w_\xi, \theta, w_{\xi\eta}\}$; $\bar{\mathbf{u}}_\ell = \{u_\ell, v_\ell, w_\ell, w_{\xi\ell}, \theta_\ell, w_{\xi\eta\ell}\}$; $\theta = -v + w_\eta$; the matrix of the elastic constants

$$\mathbf{D} = \begin{vmatrix} B_{11} & B_{12} & 0 & & & \\ B_{12} & B_{11} & 0 & & 0 & \\ 0 & 0 & B_{33} & & & \\ & & & D_{11} & D_{12} & 0 \\ & 0 & & D_{12} & D_{11} & 0 \\ & & & 0 & 0 & D_{33} \end{vmatrix}, \quad \begin{aligned} B_{11} &= Eh/(1-\nu^2), \\ B_{12} &= \nu B_{11}, \\ B_{33} &= \frac{1}{2}(1-\nu) B_{11}, \\ D_{11} &= Eh^3/12(1-\nu^2), \\ D_{12} &= \nu D_{11}, \\ D_{33} &= \frac{1}{2}(1-\nu) D_{11}. \end{aligned}$$

Summing the potential energies of the individual elements, we find the total potential energy of the shell.

2. Equations of Equilibrium of the Shell. We obtain the system of nonlinear algebraic equations of equilibrium of the shell in accordance with the principle of virtual displacements $\delta\Pi = 0$. We write the variation of the potential energy of the shell

$$\delta\Pi = \sum_{i=1}^{mn} (\delta W_i - \delta A_i) = \sum_{i=1}^{mn} \left(\int_s\int \mathbf{T}^{\mathrm{T}} \delta\varepsilon ds - \delta A_i \right) = 0; \tag{2.1}$$

$$\delta A_i = \int_s\int \mathbf{q}^{\mathrm{T}} \delta\mathbf{u} ds + \int_l \mathbf{R}_c^{\mathrm{T}} \delta\mathbf{u}_c dl + \mathbf{R}_\ell^{\mathrm{T}} \delta\bar{\mathbf{u}}_\ell. \tag{2.2}$$

Inserting (1.1) into (2.1), we have

$$\sum_{i=1}^{mn} \left(\int_s\int \mathbf{T}^{\mathrm{T}} (\delta\varepsilon_l + \delta\varepsilon_0 + \delta\varepsilon_n) ds - \int_s\int \mathbf{q}^{\mathrm{T}} \delta\mathbf{u}\, ds - \int_l \mathbf{R}_c^{\mathrm{T}} \delta\mathbf{u}_c dl - \mathbf{R}_\ell^{\mathrm{T}} \delta\bar{\mathbf{u}}_\ell \right) = 0. \tag{2.3}$$

Representing ε_n and ε_0 in the form

$$\varepsilon_n = \frac{1}{2} \bar{\mathbf{B}}\bar{\varepsilon}, \quad \delta\varepsilon_n = \bar{\mathbf{B}}\delta\bar{\varepsilon}, \quad \varepsilon_0 = \bar{\mathbf{B}}^0\bar{\varepsilon}, \quad \delta\varepsilon_0 = \bar{\mathbf{B}}^0\delta\bar{\varepsilon}, \tag{2.4}$$

where

$$\bar{\mathbf{B}}^{\mathrm{T}} = \begin{vmatrix} w_\xi & 0 & w_\eta \\ 0 & w_\eta & w_\xi \end{vmatrix}; \quad \bar{\mathbf{B}}^{0\mathrm{T}} = \begin{vmatrix} w_\xi^0 & 0 & w_\eta^0 \\ 0 & w_\eta^0 & w_\xi^0 \end{vmatrix}; \quad \bar{\varepsilon} = \{w_\xi, w_\eta\};$$

$$\bar{\varepsilon} = \mathbf{P}^*\mathbf{B}^{-1}\bar{\mathbf{u}}; \quad p_{1j}^* = (p_{3j})_\xi; \quad p_{2j}^* = (p_{3j})_\eta - (p_{2j}) \quad (j = 1, \ldots, 24);$$

p_{ij}^* are elements of the matrix $\mathbf{P}^*$; p_{ij} are elements of the matrix $\mathbf{P}$, with allowance for (1.1) and (1.2) we write (2.3) as

$$\sum_{i=1}^{mn} ((\mathbf{K}_u + \mathbf{K}_1 + \mathbf{K}_1^{\mathrm{T}} + \mathbf{K}_2 + 2\mathbf{K}_2^{\mathrm{T}} + \mathbf{K}_3)\bar{\mathbf{u}} - \mathbf{Q} - \mathbf{Q}_c - \mathbf{Q}_\ell) = 0.$$

Here,

$$\mathbf{K}_u = (\mathbf{B}^{-1})^{\mathrm{T}} \int_s\int \mathbf{P}^{\mathrm{T}}\mathbf{A}_l^{\mathrm{T}}\mathbf{D}\mathbf{A}_l\mathbf{P} ds\mathbf{B}^{-1}; \quad \mathbf{Q} = \int_s\int \mathbf{q}^{\mathrm{T}}\mathbf{P}\, ds\mathbf{B}^{-1};$$

$$\mathbf{K}_1 = (\mathbf{B}^{-1})^{\mathrm{T}} \int_s\int \mathbf{P}^{\mathrm{T}}\mathbf{A}_l^{\mathrm{T}}\mathbf{D}^*\bar{\mathbf{B}}^0\mathbf{P}^*\, ds\mathbf{B}^{-1}; \quad \mathbf{Q}_c = \int_l \mathbf{R}_c^{\mathrm{T}}\mathbf{P}_c\, dl\mathbf{B}^{-1};$$

$$\mathbf{K}_2 = \frac{1}{2} (\mathbf{B}^{-1})^{\mathrm{T}} \int_s\int \mathbf{P}^{\mathrm{T}}\bar{\mathbf{A}}_l^{\mathrm{T}}\mathbf{D}^*\bar{\mathbf{B}}\mathbf{P}^*\, ds\,\mathbf{B}^{-1}; \quad \mathbf{Q}_\ell = \mathbf{R}_\ell;$$

$$\mathbf{K}_3 = (\mathbf{B}^{-1})^{\mathrm{T}} \int_s\int (\mathbf{P}^*)^{\mathrm{T}} [\bar{\mathbf{T}}] \mathbf{P}^* ds \mathbf{B}^{-1}; \quad [\bar{\mathbf{T}}] = (\bar{\mathbf{B}}^0)^{\mathrm{T}} \mathbf{D}^* \bar{\mathbf{B}}^0 +$$

$$+ (\bar{\mathbf{B}}^0)^{\mathrm{T}} \mathbf{D}^* \bar{\mathbf{B}} + (1/2) \bar{\mathbf{B}}^{\mathrm{T}} \mathbf{D}^* \bar{\mathbf{B}}^0 + (1/2) \bar{\mathbf{B}}^{\mathrm{T}} \mathbf{D}^* \bar{\mathbf{B}};$$

$$\bar{\mathbf{A}}_l^{\mathrm{T}} = \begin{vmatrix} (\ )_\xi & 0 & (\ )_\eta \\ 0 & (\ )_\eta & (\ )_\xi \\ 0 & (\ ) & 0 \end{vmatrix}; \quad \mathbf{D}^* = \begin{vmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{11} & 0 \\ 0 & 0 & B_{33} \end{vmatrix};$$

$$(p_c)_{1j} = p_{1j};\ (p_c)_{2j} = p_{2j};\ (p_c)_{3j} = p_{3j};\ (p_c)_{4j} = (p_{3j})_\xi;$$
$$(p_c)_{5j} = -p_{2j} + (p_{3j})_\eta;\ (p_c)_{6j} = (p_{3j})_{\xi\eta}\ (j = 1, \ldots, 24);$$

$(p_c)_{ij}$ and p_{ij} are elements of the matrices $\mathbf{P}_c$ and $\mathbf{P}$, respectively. Considering the displacement compatibility conditions in accordance with [2] and the boundary conditions, we find the system of algebraic equations of equilibrium of the shell

$$\psi(u) = \bar{\mathbf{K}}\mathbf{u} - \bar{\mathbf{Q}} = 0, \tag{2.5}$$

where $\bar{\mathbf{K}}$ (the band-type stiffness matrix of the shell) is obtained by summation of the elements of the matrix $(\mathbf{K}_u + \mathbf{K}_1 + \mathbf{K}_1^{\mathrm{T}} + \mathbf{K}_2 + 2\mathbf{K}_2^{\mathrm{T}} + \mathbf{K}_3)$ with the use of the index matrix [3]; $\mathbf{Q}$ (the vector of the generalized nodal forces of the shell) is determined by adding the elements of the vector $(\mathbf{Q} + \mathbf{Q}_c + \mathbf{Q}_\ell)$ with the use of the index matrix; $\mathbf{u}$ is the vector of the nodal displacements of the shell.

3. Algorithm for Solving Shell Equilibrium Equations. To solve system (2.5), we will use the Newton–Kantorovich method [3]:

$$\frac{\partial\psi(\mathbf{u}^n)}{\partial\mathbf{u}^n}\Delta = -\psi(\mathbf{u}^n) \tag{3.1}$$

(Δ is the vector of the increments of the nodal displacements). The derivative $\partial\psi(\mathbf{u}^n)/\partial\mathbf{u}^n$ is the matrix of the second derivatives of the potential energy of the shell and is found from the second variation of potential energy in the form

$$\frac{\partial\psi(\mathbf{u}^n)}{\partial\mathbf{u}^n} = \delta^2 W = \sum_{i=1}^{mn}\int\int(\delta\mathbf{T}^{\mathrm{T}}\delta\varepsilon + \mathbf{T}^{\mathrm{T}}\delta^2\varepsilon)\,ds. \tag{3.2}$$

Using the relation $\delta^2\varepsilon = \delta^2\varepsilon_n = \delta\bar{\mathbf{B}}\delta\bar{\varepsilon}$, $\mathbf{T}^{\mathrm{T}}\delta^2\varepsilon = \delta\bar{\varepsilon}^{\mathrm{T}}\mathbf{T}^*\delta\bar{\varepsilon}$, where

$$\mathbf{T}^* = \begin{vmatrix} T_1 & T_3 \\ T_3 & T_2 \end{vmatrix},$$

with allowance for (1.2) and (2.4) we have

$$\delta^2 W = \sum_{i=1}^{mn}(\mathbf{K}_u + \mathbf{K}_4 + \mathbf{K}_4^{\mathrm{T}} + \mathbf{K}_5). \tag{3.3}$$

Here

$$\mathbf{K}_4 = (\mathbf{B}^{-1})^{\mathrm{T}}\int_s\int\mathbf{P}^{\mathrm{T}}\bar{\mathbf{A}}_l^{\mathrm{T}}\mathbf{C}\mathbf{P}^*\,ds\mathbf{B}^{-1};\ \mathbf{K}_5 = (\mathbf{B}^{-1})^{\mathrm{T}}\int_s\int(\mathbf{P}^*)^{\mathrm{T}}\mathbf{T}'\mathbf{P}^*\,ds\mathbf{B}^{-1};$$

$$\mathbf{T}' = (\bar{\mathbf{B}}^0)^{\mathrm{T}}\mathbf{D}^*\bar{\mathbf{B}}^0 + (\bar{\mathbf{B}}^0)^{\mathrm{T}}\mathbf{D}^*\bar{\mathbf{B}} + \bar{\mathbf{B}}^{\mathrm{T}}\mathbf{D}^*(\bar{\mathbf{B}}^0) + \bar{\mathbf{B}}^{\mathrm{T}}\mathbf{D}^*\bar{\mathbf{B}} + \mathbf{T}^*;$$

$$\mathbf{C} = \mathbf{D}^*\bar{\mathbf{B}}^0 + \mathbf{D}^*\bar{\mathbf{B}}.$$

Inserting (3.2) into (3.1) and allowing for (2.5), (3.3), the boundary conditions, and compatibility conditions [2], we obtain

$$\bar{\mathbf{H}}\Delta = \bar{\mathbf{Q}} - \bar{\mathbf{K}}(\mathbf{u})^n \tag{3.4}$$

($\bar{\mathbf{H}}$ is the band-structured Hessian of the system, which is determined by summation of the elements of the matrix $\mathbf{H} = \mathbf{K}_u + \mathbf{K}_4 + \mathbf{K}_4^{\mathrm{T}} + \mathbf{K}_5$).

Augmenting (3.2) with the equation

$$(\mathbf{u})^{n+1} = (\mathbf{u})^n + \Delta, \tag{3.5}$$

we seek the solution of the system of nonlinear algebraic equilibrium equations in the following manner. We assign a small value to the load parameter. As the zeroth approximation

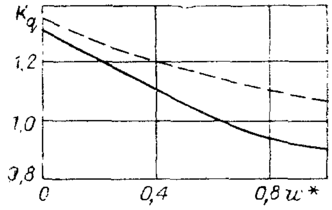

Fig. 1

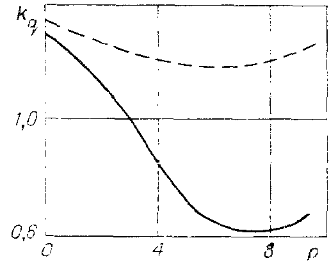

Fig. 2

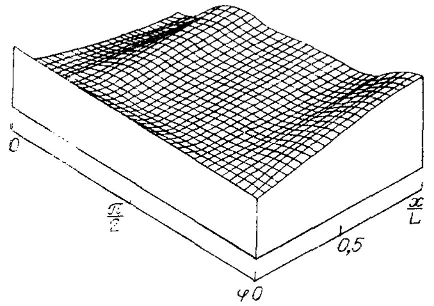

Fig. 3

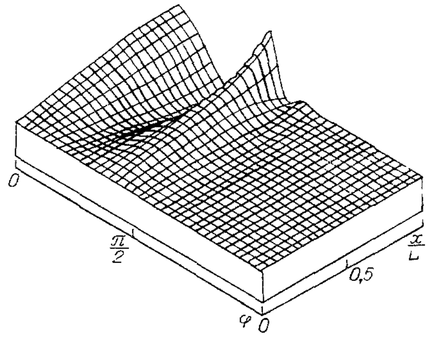

Fig. 4

we take the solution of the linear problem. We perform the iteration by scheme (3.4)-(3.5), in which $\bar{H}$ is computed once after the first iteration and remains unchanged for the other iterations. The load is then increased, and as the zeroth approximation we take the solution from the previous load level. The iteration process is performed again. In each iteration, the system of linear algebraic equations is solved by the Kraut method using the expansion $\bar{H} = L^T DL$ [4]. Having solved system (2.5), we find all of the components of the nonlinear initial stress—strain rate.

4. Stability of the Shell. To evaluate the stability of the initial state of the shell, we adopt an energy criterion of stability. In accordance with this criterion, the equilibrium state is stable if $\delta^2\Pi > 0$ and unstable if $\delta^2\Pi < 0$. It follows from this that, in conformity with the Sylvester criterion, a stable state requires that the matrix $\bar{H}$ be positive-definite. This in turn means that all of the minors of $\bar{H}$ are positive. A change in the sign of a minor is equivalent to a change in the sign of an element of the matrix D in the expansion $L^T DL$ of the matrix $\bar{H}$. The latter is easily checked in the computing algorithm without the expenditure of additional machine time.

After we find the value of the load parameter for which the equilibrium state is unstable, we seek the mode of loss of stability of the shell from the solution of the system $\bar{H}\delta = 0$. To do this, we determine one of the linearly dependent rows of the matrix $\bar{H}$, for which the minor becomes negative. This line and the corresponding column of the matrix $\bar{H}$ take zero values. Unity is inserted for the diagonal element, while the column, multiplied by the assigned displacement, is moved to the right side. The algorithm was realized in an application package written for a BÉSM-6 computer and makes it possible to take a standard approach toward the study of the stability of imperfect shells with different loads and boundary conditions.

5. Example. We studied the stability of a flowed circular cylindrical shell hinged at its end and subjected to nonaxisymmetric externalpressure, changing by the law $q = q_0(1 + \cos\varphi)$. The shell had $\bar{L} = L/R = 2$, $R/h = 100$, while the initial flaws $w^0 = (w\cos\pi x/L)\cos p\varphi$.

Figure 1 shows the dependence of the dimensionless parameter $k_q = q/q_h$ on $w^* = \bar{w}/h$ for $p = 2$, where q is the critical amplitude of the nonuniform pressure and q_h is the higher uniform critical pressure [5]. The solid lines in this and other figures correspond to the solution obtained with a nonlinear initial stress—strain state. The dashed lines show the solution obtained with a linear initial state. It is evident that the value of k_q decreases with an increase in the amplitude of the initial deflection for both the linear and nonlinear

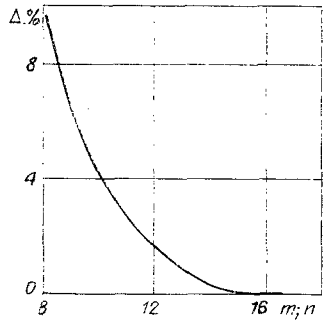

Fig. 5

initial stress–strain states. With an increase in w^*, the effect of nonlinearity of the initial state increases and reaches 20% at $w^* = 1$.

Figure 2 shows the dependence of k_q on p, characterizing the change in the initial deflection in the circumferential direction for $w^* = 0.4$. The solutions obtained with linear and nonlinear initial stress–strain states qualitatively coincide, i.e., an increase in p is initially accompanied by a decrease in the parameter k_q. The latter reaches a minimum at $p = p^*$ (p^* is the wave-formation parameter with uniform external pressure). The parameter k_q then increases, and the greatest effect on nonlinearity is seen at $p = p^*$. Here, the effect reaches 100%.

Figures 3 and 4 show the mode of deformation of the shell in the initial state and the mode of loss of stability for $p = 2$ and $w^* = 0.4$. It is evident that they do not coincide. The shell becomes unstable in its upper part, with the formation of six distinct longitudinal folds. In light of the planar symmetry of the shell and the load, we isolated one-fourth of the shell and subdivided it into n curvilinear rectangular finite elements of natural curvature in the longitudinal and transverse directions.

Figure 5 shows the graph of the convergence of the solution (the determination of the critical load) according to the number of finite elements n. The satisfactory convergence of the solution is evident. The results of the calculation were compared with the solution obtained on a 16 × 16 grid. The algorithm developed here makes it possible to use a standard approach toward studing the stability of shells with longitudinal loads and initial deflections. Here, the mode of instability is not connected with the form of the initial deflections.

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